Credit Research Initiative
Risk Management Institute, National University of Singapore

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Addendum 3 to the CRI Technical Report (Version: 2017, Update 1)

This addendum describes the technical details concerning the CRI "Probability of Default implied Ratings (PDiR)". The PDiR was introduced in 2011 to complement the high-granularity CRI Probability of Default (PD) for one-year prediction horizon by assigning a letter grade to each firm in reference to an average annual realized default rate of the Standard & Poor's (S&P) global credit rating pool. The methodology is revised and implemented on December 15th, 2017 to provide a better match to the average annual realized default rates of the S&P global rating pool over a 20-year period. The revised PDiR methodology also provides a plus/minus modifier when appropriate.

Table 1 presents the CRI PDiR mapping table as of December 15th, 2017. For example, a firm having its CRI 1-year PD in the range between 0 and 0.74 bps can be understood as a firm with creditworthiness similar to a representative S&P AAA rated firm. The PDiR boundary values will be updated periodically to incorporate significant changes in the corporate credit markets.

Table 1. 1-year PD to PDiR mapping table
As of December 15th, 2017

Rating Category		Observed S&P Average Default Rate (bps)	Smoothed S&P Average Default Rate (bps)	CRI PD lower bound (in bps)	CRI PD upper bound (in bps)
AAA		0.00	0.29	0.00	0.74
AA+		0.00	1.10	0.74	1.56
AA		2.20	1.71	1.56	1.92
AA-		3.85	2.65	1.92	3.49
A+		6.00	4.12	3.49	5.04
A		7.90	6.39	5.04	7.57
A-		7.20	9.92	7.57	13.34
BBB+		11.60	15.40	13.34	17.81
BBB		18.50	23.89	17.81	28.73
BBB-		31.10	37.04	28.73	51.67
BB+		34.55	57.39	51.67	76.13
BB		52.55	88.82	76.13	107.56
BB-		105.80	137.23	107.56	182.03
B+		199.65	211.46	182.03	228.09
B		475.90	324.52	228.09	399.33
B-		846.05	494.98	399.33	637.70
CCC/C	CCC+	2719.00	748.06	637.70	835.48
	CCC		1115.34	835.48	1515.13
	CCC-		1631.16	1515.13	2012.31
	CC		2323.16	2012.31	2645.77
	C		4217.99	2645.77	10,000

METHODOLOGY

Mapping the CRI 1-year PD to the respective PDiR requires defining the upper bound for all rating categories. The boundary values of the PDiR categories are obtained by first smoothing the average realized 1-year default rates (ADRs) for different rating categories in the S&P global corporate rating pool followed by searching for the boundary values that best match the expected 1-year PD (confined within two adjacent boundary values) to the category-specific smoothed ADR for all rating categories. The conditional expectation is taken with reference to the empirical distribution of the CRI 1-year PDs taken once a year at the yearend and averaged over the same 20-year period for which the ADRs are computed.

The description of the PDiR procedure is given below.

I. Compute smoothed ADRs

S&P publishes the realized default rates for rating categories and years in their Annual Global Corporate Default Study and Rating Transitions[1] annually. The CRI computes the smoothed ADR over the most recent 20 years. The version on the date of this writing is based on data published in the S&P report for 2016.

Due to the lack of observed defaults for S&P AAA and AA+ rated firms and data on the individual CCC/C categories (CCC+, CCC, CCC-, etc), the boundary values for these categories could not be determined without first extrapolating/interpolating ADRs for these categories. Our approach is to conduct a linear regression on logit-transformed ADR for the categories with meaningful values, and through which predict ADRs for others. Specifically,

$$\text{Logit}(ADR_p) = \log\left(\frac{ADR_p}{1 - ADR_p}\right)$$

where p represents a rating category and ADR_p is the average one-year realized default rate for category p.

[1] Table 9, One-Year Global Corporate Default Rates By Rating Modifier, S&P 2016 Annual Corporate Default Study and Rating Transitions

Fig 1. Logit (ADR) and rating categories

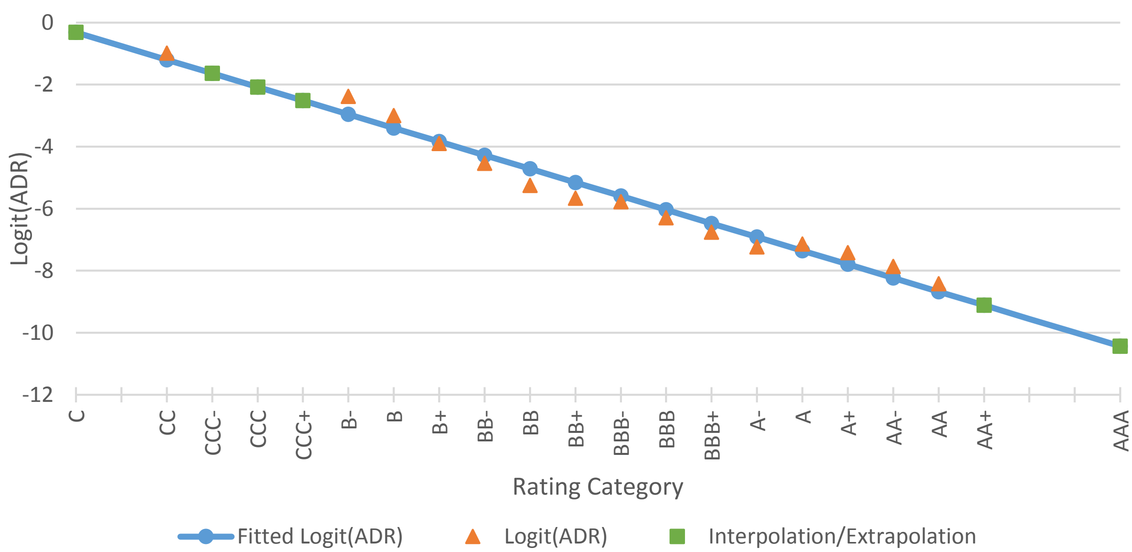


Figure 1 presents the linear regression line used to smooth the relationship between the logit-transformed ADRs and the rating categories where we have assumed the S&P reported ADR for the combined category of CCC/C can be used to represent the CC category. Note that we are only able to obtain the realized default rate for this combined category. Moreover, the spacing between AAA and AA+ is three ticks whereas that between CC and C is two ticks. These two assumptions do not affect the linear regression estimation but affect the extrapolated values. The design is motivated by the fact that without extra spacing, the PDiR would have led to a far greater number of AAA or C firms as compared to the S&P rating practice.

We then transform the fitted logit values back to probabilities to obtain the smoothed ADRs:

$$\widehat{ADR_p} = \frac{\exp\left(\widehat{\mathrm{Logit}\left(ADR_p\right)}\right)}{1 + \exp\left(\widehat{\mathrm{Logit}\left(ADR_p\right)}\right)}.$$

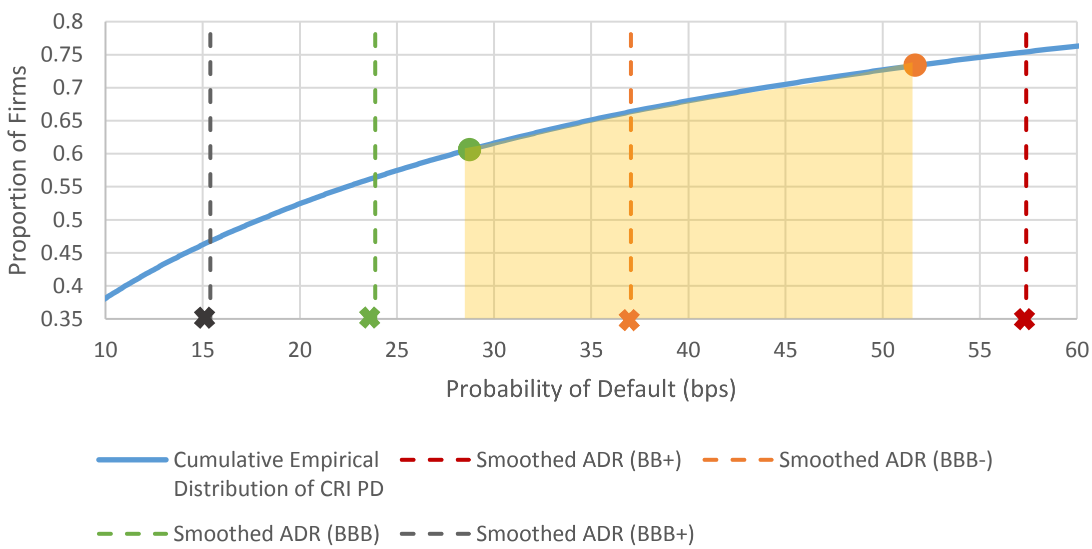
II. Find upper boundaries between different PDiR classes

The PDiR seeks to match the smoothed ADR for a specific rating category with the average probability of default (APD) using the empirical distribution of the 1-year PD in the CRI universe of exchange-listed firms. To accomplish this, we need to construct the empirical distribution and then find appropriate PD cut-off values for each rating category.

The empirical distribution is constructed with snapshots of the CRI PDs for the active firms at the yearend for each of the previous two decades. The final empirical distribution is the average of the 20 distributions. Due to the large number of exchange-listed firms and taking PDs over the 20-year period, it will need over a half-million PD points to completely characterize the empirical distribution. For computational efficiency, we are able to reduce them to around 3000 points without materially compromising quality by defining variable PD spacing (i.e., the increment between two adjacent PD values) for different ranges of PD values. Specifically, lower PD values are given more refined spacing; for example, the spacing is determined with an increment of 2×10^{-5} in empirical probability for PD less than 0.01 bps, and 1×10^{-3} for PD between 100 bps and 1000 bps.

The APD for a rating category is a conditional expected value of PD based on the empirical distribution while the PD is constrained to fall between the upper and lower boundary values for that rating category. The appropriate boundary values will then be obtained by closely matching APD to its corresponding smoothed ADR across all rating categories. Figure 2 below presents the empirical distribution of the CRI 1-year PD, and the upper/lower bounds and the smoothed ADRs for some rating categories.

Fig 2. Empirical distribution of CRI 1-year PD, boundary values, and smoothed ADR



Let $F(x)$ be the cumulative empirical distribution. Then, the conditional expected value for a rating category p with lower and upper bounds (B_{p-1} and B_p) is

$APD_p = \frac{\int_{B_{p-1}}^{B_p} x dF(x)}{F(B_p) - F(B_{p-1})}$.

Since PD only take values between 0 and 10,000 bps, the lower bound for the best performing category and the upper bound for the worst performing category are naturally set. The remaining boundary values are then selected between two adjacent smoothed ADRs by minimizing the sum of the squared relative differences between the APD and ADR for all rating categories, where the ADR is used as the base for computing the relative difference. For illustration, the yellow shaded area in Figure 2 defines the range of PDs for the BBB- category. The goal is to find boundary values so that the average PD confined to this range is as close to the smoothed ADR for the BBB- category.

Since there are 21 rating categories, 20 unknown boundary values $(B_1, B_2, \ldots, B_{20})$ are to be solved together. Note that each B_p is constrained between two adjacent smoothed ADRs. Mathematically, these 20 boundary values influencing APDs are chosen to minimize the following objective function:

$$\sum_{p \in \{AAA, AA+, \ldots, C\}} \left(\frac{APD_p - \widehat{ADR}_p}{\widehat{ADR}_p} \right)^2$$

where $\widehat{ADR}_p$ represents the smoothed ADR for rating category p. Note that the above objective function is not smooth in terms of boundary values. We thus deploy a sequential Monte Carlo algorithm to obtain the optimal solution.

III. Map PDiR to 1-year PD

Large PD variations that cause a firm to move into a different rating category are informative as they give an update on the firms' financial health. However, slight PD variations may also trigger a shift in credit rating when the firm's PD is on or near a boundary value. Naturally, one would like to avoid credit rating shifts due to a minute transitory PD variation being hardcoded into a rating change. In order to reduce those frequent senseless rating changes, the CRI assigns the PDiR by first computing a two-week moving average PD, i.e., 10 working days, and then map it according to the boundary values for different rating categories provided in Table 1.